## RADIAL DISTRIBUTION OF THE POSITION OF THE ARC IN A PLASMATRON

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The radial-distribution function of the position of the arc in a plasmatron of an axial scheme with gas-vortex stabilization has been determined from the results of standard measurements of the radial pulsations of the projection of the arc. A model allowing explanation of the displacement of the distribution maximum from the axis of the discharge chamber with increase in the current has been proposed. Based on this model, the condition for the discharge-channel radius with which gas-vortex stabilization is efficient has been obtained.

Introduction. It is well known that the region of interaction of the arc in the channel of a plasmatron of an axial scheme with the gas flow can be subdivided into two parts [1]: in the first part, gas flow is nearly laminar and the arc, being located along the channel axis due to the gas-vortex stabilization, has a stable shape; in the second part, we have a sharp turbulization of the flow due to the mixing of a cold-wall layer of the gas with the thermal layer of the arc. This leads to a stability loss and the appearance of considerable radial pulsations of the arc. Experimental investigation of these pulsations has been carried out in many works [1-4]. The experimental scheme remained constant for the most part: pulsations of the projection of a point of the arc in a fixed cross section onto the axis perpendicular to the channel was measured through the orifice of the plasmatron chamber. Different statistical characteristics of the arc, on whose basis conclusions on its structure were drawn, were measured in such a manner.

We would like to call attention to the possibility of obtaining additional information on the position of the arc in the channel, namely, the radial-distribution function of the position of the arc, from experiments of this type. Such information can be employed in calculating the geometry of the discharge channel of a plasmatron.

Experimental Procedure. The measurements were carried out in a single-chamber sectional plasmatron with gas-vortex arc stabilization. The diameter of the discharge channel was equal to $10^{-2} \mathrm{~m}$; the distance between the electrodes was $10^{-1} \mathrm{~m}$. Lateral vibrations of the arc were measured at a distance of $1.5 \cdot 10^{-2} \mathrm{~m}$ from the outlet. The arc current varied from 100 to 200 A ; the flow rate of the gas remained constant and equal to $0.006 \mathrm{~kg} / \mathrm{sec}$.

Observations of the arc were carried out through a narrow transverse slot in the interelectrode insert closed by an organic-glass plate on the outside. Using the optical system, we formed the image of the luminous point of the arc in the form of a narrow strip located across the slot. The light signal arrived at the photodiode surface through a triangular aperture with a small angle of opening and thus an electric signal (proportional to the transverse displacement of the arc's point) was generated. The signal was digitized with the use of an analog-to-digital converter and was fed into a computer. Its further processing was carried out in the environment of an integrated Mathematica package.

Figure 1 gives a typical digital "oscillogram" of the signal, whereas Fig. 2 gives the probability densities obtained for the modulus of the projection of the position of the arc's point. In what follows, the probability density will be referred to as the distribution function.

Mathematical Model. The position of the arc in the channel cross section can be prescribed by the cylindrical coordinates $r$ and $\varphi$. These coordinates are independent, and their joint distribution function can be represented in the form of the product of two distribution functions - by radius and by angle

$$
F(r, \varphi)=R(r) \Phi(\varphi)
$$

By virtue of the axial symmetry of the problem, the angular distribution will be uniform, i.e.,

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Fig. 1. Typical digital oscillogram of the signal.
Fig. 2. Distribution of the modulus of the projection of the position of the arc's point onto the lateral axis $X(|x|)(1 / \mathrm{m})$ for different currents: 1) $I=100$; 2) $150 ; 3) 200 \mathrm{~A}$.

$$
\Phi(\varphi)=\frac{1}{2 \pi} .
$$

Then, in the Cartesian coordinates, the joint distribution function for $x$ and $y$ can be written as

$$
g(x, y)=F(r, \varphi)\left|\frac{D(r, \varphi)}{D(x, y)}\right|=F(r, \varphi)\left(\left|\frac{D(x, y)}{D(r, \varphi)}\right|\right)^{-1}
$$

where

$$
\left|\frac{D(r, \varphi)}{D(x, y)}\right|=\left(\left|\frac{D(x, y)}{D(r, \varphi)}\right|\right)^{-1}=\left(\left|\begin{array}{ll}
\frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} \\
\frac{\partial x}{\partial \varphi} & \frac{\partial y}{\partial \varphi}
\end{array}\right|\right)^{-1}=\frac{1}{r}
$$

is the modulus of the Jacobian of transformation [5].
Thus, we have

$$
g(x, y)=\frac{1}{2 \pi} \frac{R(r)}{r}=\frac{1}{2 \pi} \frac{R\left(\sqrt{x^{2}+y^{2}}\right)}{\sqrt{x^{2}+y^{2}}} .
$$

We obtain the distribution along $x$, having integrated the distribution $g(x, y)$ over all the values of $y$ permissible for a fixed $x$, i.e., going from $-\sqrt{a^{2}-x^{2}}$ to $\sqrt{a^{2}-x^{2}}$ :

$$
X(x)=\int_{-\sqrt{a^{2}-x^{2}}}^{\sqrt{a^{2}-x^{2}}} \frac{1}{2 \pi} \frac{R\left(\sqrt{x^{2}+y^{2}}\right)}{\sqrt{x^{2}+y^{2}}} d y=2 \int_{0}^{\sqrt{a^{2}-x^{2}}} \frac{1}{2 \pi} \frac{R\left(\sqrt{x^{2}+y^{2}}\right)}{\sqrt{x^{2}+y^{2}}} d y .
$$

Passage to a new integration constant $\left(r=\sqrt{x^{2}+y^{2}}\right)$ enables us to rewrite the expression for $X(x)$ as follows:

$$
\begin{equation*}
X(x)=\int_{|x|}^{a} \frac{1}{\pi} \frac{R(r)}{\sqrt{r^{2}-x^{2}}} d r . \tag{1}
\end{equation*}
$$



Fig. 3. Radial-distribution functions of the arc $R(r)(1 / \mathrm{m})$ for different currents: $1-3)$ notation is the same as in Fig. 2.

Since $X(x)=X(-x)$, we have $X(x)=0.5 X(|x|)$. The distribution function $X(|x|)$ is known from the experiment. Thus, the integral equation (1) obtained enables us to find the radial distribution of the position of the arc in the plasmatron's discharge channel.

Equation (1) can be rewritten in the standard form

$$
X(x)=\int_{x}^{a} K(x, r) R(r) d r, \quad x \geq 0
$$

where

$$
K(x, r)=\frac{2}{\pi} \frac{1}{\sqrt{r^{2}-x^{2}}}
$$

is the kernel of the integral equation.
Equation (1) has been solved numerically by the method of replacement of the integral by a quadrature sum [6]. Figure 3 plots the solutions for the distribution functions of the projections shown in Fig. 2.

Discussion of the Results. It is seen that the curves are nearly symmetric about the value of the radius corresponding to the probability maximum and are nearly Gaussian in shape. The position of the probability maximum shifts in the direction of larger distances to the axis as the arc current increases. The width of the distribution function increases only slightly.

As the distance to the axis decreases, the probability of finding the arc decreases, tending to zero. This means that the regular component of the motion of the arc on the oscillogram of Fig. 1 cannot be caused by the vibrations of the arc relative to the channel axis and, apparently, is related to the rotational motion of the arc about the axis due to the tangential component of the velocity of the blown-gas flow.

The displacement of the arc from the chamber axis with increase in the current strength can qualitatively be explained as follows. Due to the lateral turbulent pulsations of the gas velocity, the arc takes a spiral shape and rotates about the axis together with the gas flow. It creates an axial magnetic field B proportional to the current strength. An Ampere force directed along the chamber radius from the axis and forcing out the arc to the chamber walls acts on the arc element $d l$ on the source side of this field. On the other hand, a "buoyancy" force equal to $\left(\rho v^{2} / r\right) S d l$ acts on the arc element due to the pressure gradient in the gas in the direction to the chamber axis. Since the density of the plasma in the arc is much lower than the density of the blown gas, we can disregard the mass of the arc element. Hence we obtain the condition of equilibrium of the arc element

$$
\begin{equation*}
I B d l=\rho \frac{v^{2}}{r} S d l \tag{2}
\end{equation*}
$$



Fig. 4. Current strength $I(\mathrm{~A})$ vs. $f(\tilde{r})$.
For the gasdynamic method of twisting of the gas in the discharge chamber of a plasmatron of radius $a$, we can describe the tangential velocity of the gas as a function of the axis of the discharge channel by the following empirical formula consistent with experimental data [7]:

$$
v(r)=A \frac{1-\exp \left(-k r^{2}\right)}{r}
$$

where the constants $A$ and $k$ are related to the maximum velocity $v_{\mathrm{m}}$ and its coordinate $r_{\mathrm{m}}$ by

$$
A=1.4 r_{\mathrm{m}} v_{\mathrm{m}}, \quad k=1.25 / r_{\mathrm{m}}^{2}, \quad r_{\mathrm{m}}=a / \sqrt{3}
$$

Taking into account that $B \sim I$ and $S \sim I$ as a first approximation, from (2) we obtain

$$
\begin{equation*}
I=\alpha f(\tilde{r}) \tag{3}
\end{equation*}
$$

here $f(\tilde{r})=\left[1-\exp \left(-1.25 \tilde{r}^{2}\right)\right]^{2} / \tilde{r}^{3}, \tilde{r}=r / r_{\mathrm{m}}$, and the value of the coefficient $\alpha$ is proportional to $v_{\mathrm{m}}^{2} / a$.
Figure 4 gives experimental data on the dependence of the current on $f(\tilde{r})$, where the quantity $\tilde{r}$ corresponds to the position of the maxima of the radial-distribution function of the position of the arc.

It is seen that the dependence is nearly proportional (the correlation coefficient is equal to 0.9 ). This enables us to infer that the model proposed qualitatively correctly explains the effect of deviation of the arc from the channel axis as the current increases.

Evaluation of the coefficient $\alpha$ in formula (3) yields a value of $\alpha=315$. The maximum of the tangential velocity for experimental conditions was $145 \mathrm{~m} / \mathrm{sec}$. The maximum of the function $f(\tilde{r})$ is attained for $\tilde{r}=0.66$ and is equal to 0.62 . This enables us to obtain, from (3), the condition for the discharge-channel radius, with which gas-vortex stabilization is effective:

$$
a \leq\left(\frac{v_{\mathrm{m}}}{145}\right)^{2} \frac{1}{I}
$$

The relation obtained can turn out to be useful in calculating and designing plasmatrons with gas-vortex stabilization of the position of the arc.

## CONCLUSIONS

We have proposed a procedure for determination of the radial distribution function of the position of the arc in a plasmatron with gas-vortex stabilization from the results of measurement of the pulsations of the projection of the position of the arc's point onto the lateral axis. It has been shown that the existing effect of displacement of the mean position of the arc from the axis of the discharge chamber with increase in the arc current is attributable to the interaction between the arc rotating in a twisted gas flow with a strong turbulence and the intrinsic magnetic field. Based on the model proposed, we have obtained a relation between the radius of the discharge channel, the strength of the
arc current, and the maximum tangential velocity of the gas; this relation determines the condition of efficiency of gasvortex stabilization.

## NOTATION

$A=1.4 r_{\mathrm{m}} \nu_{\mathrm{m}}$, parameter of the function of the radial dependence of the tangential gas velocity; $a$, radius of the discharge channel, $\mathrm{m} ; B$, induction of the magnetic field, $\mathrm{T} ; d l$, element of the arc length, $\mathrm{m} ; F(r, \varphi)$, joint distribution function, $1 / \mathrm{m} ; f(\widetilde{r})$, function of the dimensionless coordinate; $g(x, y)$, joint distribution function, $1 / \mathrm{m}^{2} ; I$, arc-current strength, A; $K(x, r)$, kernel of the integral equation; $k=1.25 / r_{\mathrm{m}}^{2}$, parameter of the function of the radial dependence of the tangential gas velocity; $r$ and $\varphi$, cylindrical coordinates of the arc's point, m and rad; $R(r)$, radial distribution function, $1 / \mathrm{m} ; \tilde{r}=r / r_{\mathrm{m}}$, dimensionless radial coordinate; $S$, cross-sectional area of the arc, $\mathrm{m}^{2} ; v$, tangential gas velocity, $\mathrm{m} / \mathrm{sec} ; v_{\mathrm{m}}$ and $r_{\mathrm{m}}$, maximum tangential velocity and its radial coordinate; $X(x)$, distribution function of the projection onto the $x$ axis, $1 / \mathrm{m} ; x$ and $y$, Cartesian coordinates, $\mathrm{m} ; \alpha$, proportionality factor; $\rho$, density of the gas, $\mathrm{kg} / \mathrm{m}^{3} ; \Phi(\varphi)$, angular distribution function. Subscript: m, maximum.

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